A continuum model to rationalise the time-dependent kinematic and stress responses of human amnion

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Introduction

Amnion, the inner, only few hundreds of micrometers thick but mechanically dominant layer of the foetal membrane is characterised by exceptional material characteristics. This includes drastic lateral contractility upon uniaxial loading both within and out of plane [1, 2]. These effects are time-dependent and further experiments reveal pronounced stress relaxation but only little creep [2]. In the present contribution, uniaxial and biaxial experimental data is shown and a modified version of the constitutive model in [1, 3] is presented that is able to rationalise these results.

Modelling

Motivated by [2] the model accounts for two visco-elastic mechanisms with two distinct time scales: Fast volumetric changes along with rotation of collagen fibres and long-term creep of the fibres. The latter are modelled by N representative, mechanically equivalent fibre families uniformly distributed in-plane and with a small out-of-plane inclination, embedded in a compressible matrix.

Denoting by $\mathbf{F}$ the deformation gradient, and the current fibre direction and stretch by the vectors $\boldsymbol{m}_{\mathrm{e}}^{i}$ with length $\lambda_{\mathrm{e},i}$, the free energy of the suggested Rubin-Bodner-type model reads

$$\Psi = \frac{\mu_0}{2q}(e^{qg} - 1), \quad g = g_1(J_{\mathrm{e}}) + g_2(I_1, J) + g_3(\lambda_{\mathrm{e},i}),\ i = 1, 2, ..., N, \quad I_1 = \mathrm{tr}(\mathbf{F}^{\mathrm{T}}\mathbf{F}),\ J = \det \mathbf{F} \tag{1}$$

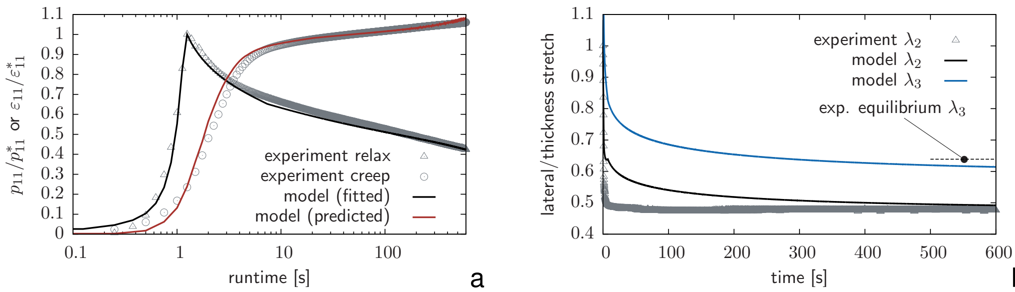
where μ_0, q are material parameters (cf. [3]). The kinematic variables J_{e} and $\lambda_{\mathrm{e},i}$ account for viscoelasticity and result from evolution equations. With $\mathbf{l} = \dot{\mathbf{F}}\mathbf{F}^{-1}$, $\mathbf{d} = \frac{1}{2}(\mathbf{l} + \mathbf{l}^{\mathrm{T}})$ these take the form

$$\dot{J}_{\mathrm{e}} = J_{\mathrm{e}}\,\mathrm{tr}\mathbf{d} - \Gamma_{\mathrm{M}}(J_{\mathrm{e}}), \quad \dot{\boldsymbol{m}}_{\mathrm{e}}^{i} = \mathbf{l}\boldsymbol{m}_{\mathrm{e}}^{i} - \Gamma_{\mathrm{F}}(\lambda_{\mathrm{e},i})\boldsymbol{m}_{\mathrm{e}}^{i},\ i = 1, 2, ..., N, \tag{2}$$

where Γ_{M} and Γ_{F} determine the rates of dissipation. These functions as well as g_1, g_2, g_3 were specified in line with classical formulations in tissue biomechanics. Adequate algorithms to integrate Eqns. (2) were implemented and parameters were determined by comparison with experimental data.

Results and discussion

Simulations of the uniaxial tests [2] indicate that the model excellently captures the normalised tension and strain data (Fig. 1a) and agrees qualitatively with the kinematic relaxation response (Fig. 1b).



a b

Fig. 1: Comparison of the model with uniaxial relaxation and creep data [2].

A Rubin-Bodner type model has already successfully been used in [1] to account for the quasi-static tension and in-plane kinematic response of human amnion. Here, we show that an advanced model is also able to account for the peculiar out-of plane and time-dependent behaviour.

References

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